

**ANALYTICAL STUDY FOR THE BOUNDARY LAYER FLOW IN  
THE PRESENCE OF HEAT TRANSFER THROUGH A POROUS  
MEDIUM**

**Ram Naresh Singh**

*Assistant Professor, Department of Mathematics, Sardar Patel Mahavidyalaya,  
Chandrapur (M.H.), Email-ramnaresh\_s@yahoo.co.in*

**Paper Received On:** 21 JUNE 2021

**Peer Reviewed On:** 30 JUNE 2021

**Published On:** 1 JULY 2021

**Content Originality & Unique:** 70%

---

**Abstract**

---

*In this paper we study a problem of the boundary layer flow through a porous media in the presence of heat transfer. Here we consider high porosity bounded by a semi-infinite horizontal plate. The main aim of this study is to point out local similarity transformations for the boundary layer flow, through a homogeneous porous medium. Here we applying finite difference schemes to find out the numerical solutions of the problem. The free stream velocity and the temperature far away from the plate are exponential function of variables.*

**Keywords:-** *Infinite Plate, Porous Medium, Finite Difference Scheme.*



*Scholarly Research Journal's* is licensed Based on a work at [www.srjis.com](http://www.srjis.com)

**Introduction:-**

The problem of the boundary layer flow through porous medium in the presence of heat transfer with the use of local similarity transformations has been investigated. Patankar [1980] have investigated the effect of fluid flow past an hemisphere with heat transfer. Neild and Bejan [1998] is studied the effect of the convection in porous media. Yang and Chang [2000] have studied the flow and heat transfer in a curved pipe with periodically varying curvature. The free stream velocity is constant and the temperature far away from the plate are exponential function of variable. Acharya and Singh (2000) studied the effect of magnetic field on the force convection and mass transfer flow through porous medium with constant suction and constant

heat flux. Ahmed, Sharma et. al (2005) discussed free convective MHD flow and heat transfer through porous medium between two long way walls. Ahmed and Sharma (1997) analysed three dimensional free convective flow of an incompressible viscous fluid through a porous medium with uniform free stream velocity. Ambedkar and Rai (2004) presented a problem on numerical solution of free convection effects of MHD Stoke's problem. Chiiti and Prasad (2006) analysed free convection flow of heat and mass transfer past a vertical porous plate. Ferdows et. al (2005) discussed similarity solution for MHD flow through vertical porous plate with suction. Jaiswal and Soundalgekar (2005) investigated transient forced and free convection flow of dissipative fluid with mass transfer past an infinite vertical plate with constant heat flux. Kumari and Nath (2004) discussed transient MHD rotating flow over a rotating sphere in the vicinity of the equator. Raju et. al (1984) analysed a formulation of combined force and free convection past horizontal and vertical surface. Sattar (1992) studied free and forced convection flow through a porous medium near the leading edge. Sattar (1993) presented a free and forced connection boundary layer flow through a porous medium with large suction. Singh and Dikshit (1988) discussed hydro magnetic flow past a continuously moving semi-infinite plate for large suction. Singh et. al (2008) analysed computational study of hydro magnetic effects on the viscous in compressible dissipative fluid past an infinite vertical plate. Soundalgekar and Thakkar (1977) studied MHD forced and free convectional flow past a semi-infinite plate. Tomar et. al (2009) discussed a numerical study of the three dimensional coquette MHD flow through a porous medium with heat transfer.

The main purpose is to point out local similarity transformations for the boundary layer flow through a porous medium. We assuming that the porosity is bounded by a semi-infinite horizontal plate. Similarly, transformations are the transformation in which n-independent variable of the system of partial differential equations can be converted into a system with n-1 independent variables. The free stream velocity is constant a monomial or a polynomial and the temperature far away from the plate is constant. Here we use finite difference techniques to find out of the numerical solution of the problem.

### **Governing Equations**

We consider an viscous incompressible fluid through a porous medium with high porosity which is bounded by a semi infinite horizontal plate in the presence of heat transfer.

The flow is governed by the equations

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \frac{\partial^2 u}{\partial y^2} - \nu \frac{\gamma}{k} u - C\gamma^2 u^2 \quad (2)$$

$$u \frac{\partial \theta}{\partial x} + v \frac{\partial \theta}{\partial y} = \frac{C_t}{\rho C_p} \frac{\partial^2 \theta}{\partial y^2} \quad (3)$$

Where  $[C_t = (1 - \gamma)C_s + \gamma C_f]$ ,  $C_t$  is the thermal conductivity and  $C_s$  is the conductivity of the solid and  $C_f$  is the conductivity of the fluid,  $\gamma$  is the porosity,  $C$  Forchheimer's inertia coefficient,  $k$  the permeability of the porous medium,  $\theta$  is temperature of the fluid.

The boundary conditions of the problem are

$$\left. \begin{aligned} u = 0, v = 0, \theta = \theta_c, \quad \text{at } y = 0 \\ u \rightarrow U_f, \theta \rightarrow \theta_\infty, \quad \text{as } y \rightarrow \infty \end{aligned} \right\} \quad (4)$$

Where  $U_f$ , is the free stream velocity,  $\theta_c$  the constant temperature of the horizontal plate,  $\theta_c$  the constant temperature of the horizontal plate,  $\theta_\infty$  the temperature of the fluid far away from the plate.

In the free stream velocity equation (2) becomes

$$U_f \frac{dU_f}{dx} = -\frac{1}{\rho} \frac{\partial p}{\partial x} - \nu \frac{\gamma}{k} U_f - C\gamma^2 U_f^2 \quad (5)$$

Now from equation (2) and (5) we get

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + \nu \frac{\gamma}{k} (U_f - u) + C\gamma^2 (U_f^2 - u^2) + U_f \frac{dU_f}{dx} \quad (6)$$

Equations are non-dimensional by using

$$\left. \begin{aligned} x^* = \frac{U_0}{\nu} x, \quad y^* = \frac{U_0}{\nu} y, \quad u^* = \frac{u}{U_0} \\ v^* = \frac{v}{U_0}, \quad U_f^* = \frac{U_f}{U_0}, \quad \theta^* = \frac{\theta - \theta_c}{\theta_c} \end{aligned} \right\} \quad (7)$$

Substituting these variables in in equation (1), (3) and (6) (after dropping the astrics) we get

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (8)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + U_f \frac{dU_f}{dx} + \alpha (U_f - u) + \beta ((U_f^2 - u^2)) \quad (9)$$

$$u \frac{\partial \theta}{\partial x} + v \frac{\partial \theta}{\partial y} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial y^2} \quad (10)$$

Where,  $\alpha = \frac{\nu}{U_0^2 k}, \beta = \frac{\nu}{U_0}$

$$P_r = \frac{1}{c_t}$$

The corresponding boundary conditions becomes,

$$\left. \begin{aligned} u = 0, \quad v = 0, \quad \theta(0) = 0, \quad \text{at } y = 0 \\ u \rightarrow U_f, \quad \theta \rightarrow \theta_\infty(x) \quad \text{as } y \rightarrow \infty \end{aligned} \right\} \quad (11)$$

Now introducing the transformations

$$s = ye^{\left(\frac{m}{2}\right)x}, \quad u = e^{mx} f_1(s)$$

$$v = e^{\left(\frac{m}{2}\right)x} f_2(s), \quad U_f = ke^{mx}$$

$$\theta = e^{mx} f_3(s), \quad \theta_\infty = \lambda e^{mx}$$

Here,  $m, k, \lambda$  are constant

In equations (8)-(10) we get the following local similarity system of ordinary differential equations

$$\left(\frac{m}{2}\right) s f_1' + f_2' + m f_1 = 0 \quad (12)$$

$$f_1'' - m f_1^2 - \left(\frac{m}{2}\right) s f_1' f_2 + \alpha^* (k - f_1) + \beta (k^2 - f_1^2) = 0 \quad (13)$$

$$m f_1 f_3 + s f_1 f_3' \left(\frac{m}{2}\right) + f_2 f_3' = \frac{1}{P_r} f_3'' \quad (14)$$

Here primes denotes differentiation with respect to  $s$  and  $\alpha^* = \alpha/e^{mx}$ .

Now the boundary conditions (11) becomes

$$\left. \begin{aligned} f_1(0) = 0, \quad f_2(0) = 0, \quad f_3(0) = 0 \\ f_1(\infty) = k, \quad f_3(\infty) = \lambda \end{aligned} \right\} \quad (15)$$

### Result and Discussion

Here we observe the local similarity transformations for the boundary layer flow through a homogeneous porous medium ,which is bounded by a semi infinite horizontal plate in the presence of heat transfer. Now we solve the differential equations(12),(13)and (14) using numerical techniques with the boundary conditions and compare results with the solution of the differential equations (8),(9) and (10) using boundary conditions.

### References

- Patankar, S.V.(1980), "Numerical Heat transfer and fluid flow hemisphere." Washington D.C.  
 Nield, D.A. and Bejan,A.(1998), "Convection in porous media". Springer Verlag.  
 Yang,J.B.,Chang,S.F. and Wu,W.(2000), "Flow and Heat transfer in a curved pipe with periodically varying curvature."Int.Journal. Vol.27,No.1,pp.133-143.

- Acharya, D. and Singh (2000), "The effect of magnetic field on the free convection and mass transfer flow through porous medium with constant suction and constant heat flux." *Indian J. Pure Appl.*, Vol. 31., p.11-19.
- Ahamed, N., Sharma, D. and Das, V.H. (2005), "Free convective MHD flow and heat transfer through porous medium between two long way walls". *J. Raj Acad. Phy. Sci.*, Vol. 4, No. 4, pp. 253 – 269.
- Ahmed, N. and Sharma, D. (1997), "Three dimensional free convective flow of an incompressible viscous fluid through a porous medium with uniform free stream velocity". *Ind. J. Pure Appl. Math* 28(10), pp. 13 – 45.
- Ambethkar, V. and Rai, L. (2004), "Numerical solution of free convection effects of MHD stokes problem." *J. Raj. Acad. Phy. Sci.* Vol. 3, No. 4 pp. 291 – 304.
- Chitti, B.D. and Prasada, R.D.R.V. (2006), "Free convection flow of heat and mass transfer past a vertical porous plate." *Acta Ciencia Indica* Vol. XXXVI No. 2 pp. 673 – 684.
- Ferdows, M. and Ota, M. (2005), "Similarity solution for MHD flow through vertical porous plate with suction". *Journal of computational and Applied Mechanics*, Vol. 6 No. 1, pp. 15 – 25.
- Jaiswal, B.S. and Soundalgekar, V.M. (2005), "Transient forced and free convection flow of dissipative fluid with mass transfer past an infinite vertical plate with constant heat flux." *J. Raj. Acad. Phy. Sci.* Vol. 4, No. 4, pp. 311 – 323.
- Kumari, M. and Nath, G. (2004), "Transient MHD rotating flow over a rotating sphere in the vicinity of the equator". *Int. Jou. of Engineering Science*, Vol. 42 pp. 1817-1829.
- Raju, M.S. Liu, X.R. and Law, C.K. (1984), "A formulation of combined forced and free convection past horizontal and vertical surfaces. *International Journal of heat and mass transfer.*" Vol. 27(9 – 12), pp. 2215 – 2224.
- Sattar, M.A. (1992), "Free and forced convection flow through a porous medium near the leading edge". *Astrophysics and space science.* Vol. 191, pp. 323 – 328.
- Sattar, M.A. (1993), "Free and forced connection boundary layer flow through a porous medium with large suction". *International Journal of Energy Research*, Vol. 17, pp 1- 7.
- Singh and Dikshit (1988), "Hydro magnetic flow past a continuously moving semi-infinite plate for large suction." *Astrophysics and space science*, Vol. 148 pp, 249 – 256.
- Singh, R.N., Tomar, H.S. and Sharma, D.S. (2008), "Computational study of hydro magnetic effects on the viscous incompressible dissipative fluid past an infinite vertical plate". *Ultra Science.* Vol. 20(3) M, pp. 619 – 626.
- Soundalgekar, V.M. and Thakhar, H.S. (1977). "MHD forced and free convection flow past a semi-infinite plate." *AIAA Journal*, Vol. 15, pp. 457 – 459.
- Tomar, H.S., Singh, R.N. and Sharma, D.S. (2009), "A numerical study of the three dimensional coquette MHD flow through a porous medium with heat transfer". *Acta Ciencia Indica.* Vol. XXXVM, No. 3, pp. 823 – 828.